



Figure 1

The coordinates for feedbacks of this scheme are chosen that their measuring has to the least technical difficulties: angle displacement (ϕ) and velocities (ω) for executive mechanism (given on the figure by solid lines), and electromagnetic torque (T) of actuating motor. For simplicity the internal loop for torque (armature current) on the figure is shown as transfer function of the closed loop of the torque (current) F_{acl} .

Using the formula of structure conversion for transformation of the block diagrams, one can find that transfer function of system is given by

$$T(p) = N(p) / D(p). \quad (2)$$

Here

$$N(p) = F_{rp} F_{rv} F_{acl} \left[\omega_{01}^2 \omega_{03}^2 \cdot \dots \cdot \omega_{0,2n-2}^2 \right] / (J_n p^2),$$

$$D(p) = 1 + \left| \omega_{01}^2 + \omega_{02}^2 + \dots + \omega_{0,2n-2}^2 + (F_{rp} / p) F_{rv} F_{acl} \left[\omega_{01}^2 \omega_{03}^2 \cdot \dots \cdot \omega_{0,2n-2}^2 \right] / (J_n p) \right|_1 + A_2(p) + A_3(p) + \dots$$

$$+ A_{n-2}(p) + \left| \omega_{01}^2 \omega_{03}^2 \cdot \dots \cdot \omega_{0,2n-3}^2 \right|_{n-1},$$

where $\omega_{01} = \sqrt{(c_{12} / J_1)}$; $\omega_{02} = \sqrt{(c_{12} / J_2)}$; \dots ; $\omega_{0,2n-2} = \sqrt{(c_{0,2n-2} / J_n)}$ -

eigenfrequencies of the system, $| \dots |_1, A_2(p), A_3(p), \dots, A_{n-2}(p), | \dots |_{n-1}$ - single, pair, triple, \dots , $n-2$, $n-1$ discontinuous loops at even n .

One can see that the characteristic equation of system $D(p) = 0$ that the system can not be made steady in principle using only feedbacks from velocity and displacement of n -th mass of the system. As the analysis has shown, a minimal vector of coordinates measured (shown on figure 1 by dashed lines) for creation of an operable nonadaptive control system with n -mass elastic object by means of an incomplete vector of

coordinates, when it needs the performance of a presence condition of all degrees and positivity of coefficients of the characteristic equation, is represented as

- 1) $x_{k1} = (\omega_1, \omega_2, \dots, \omega_n, \varphi_n)$ at a proportional regulator of velocity;
- 2) $x_{k2} = (T_{12}, T_{23}, \dots, T_{n-1,n}, \omega_n, \varphi_n)$ at a proportional – integrated regulator of velocity.

The first case at practical realization is more preferable because the velocities of all masses are more accessible to measure than their elastic moments.

Then the characteristic equation $D(P)$ of system at x_{k1} assumes a form

$$D(p) = b_0 p^{2n} + b_1 p^{2n-1} + b_2 p^{2n-2} + \dots + b_{2n-1} p + b_{2n} = 0, \quad (3)$$

where

$$b_0 = a_0 = \chi / (F_{rv} F_{acl}); \quad b_1 = \chi / J_1; \quad b_2 = a_1 = \chi (\omega_{01}^2 + \omega_{02}^2 + \dots + \omega_{0,2n-2}^2) / (F_{rv} F_{acl}); \dots ;$$

$$b_{2n-1} = \chi [\omega_{02}^2 \omega_{04}^2 \omega_{06}^2 \dots \omega_{0,2n-4}^2 \omega_{0,2n-2}^2] / J_1; \quad b_{2n} = a_n = 1; \quad \chi = J_n / [F_{rv} (\omega_{01} \omega_{03} \omega_{05} \dots \omega_{0,2n-3})^2].$$

Using coefficients of the characteristic equation (3) and the algebraic Routh criterion of stability as algorithmically most convenient, one can formulate the table Payca for this case. On the basis of this stability criterion a system will be the stable, it is necessary and enough, if all coefficients of the equation (3) and values of the first column of the table Payca were positive. Omitting well-known procedure of construction of the Routh table, one can present stability conditions of EMS by way of system of equations as follows

$$\left. \begin{aligned} b_0 > 0, b_1 > 0, b_2 > 0, b_3 > 0, \dots, b_{2n-2} > 0, b_{2n-1} > 0, \\ C_{11} = b_0 = a_0 = \chi / (F_{rv} F_{acl}) > 0, \\ C_{21} = b_1 = \chi / J_1 > 0, \\ C_{31} = b_2 - (b_0 b_3) / b_1 = \chi [\omega_{01}^2 + \omega_{03}^2 + \dots + \omega_{0,2n-2}^2] / (F_{rv} F_{acl}) > 0, \\ C_{41} = b_3 - [b_1 (b_4 b_1 - b_0 b_5) / (b_2 b_1 + b_0 b_3)] > 0; \dots \end{aligned} \right\} \quad (4)$$

Calculation for coefficients $C_{11}, C_{21}, C_{31}, C_{41}, \dots, C_{n-1}$ can be received by means of presentation them through own frequencies of masses $\omega_{01}, \omega_{02}, \omega_{03}, \dots, \omega_{0,2n-2}$. Attempts to reduce the mathematical expressions of conditions (4) to more compact kind for n-mass system give bulky enough ones. However in practice it is necessary to deal with EMS having a mechanical part which can be represented by the system not more than sixth order, and in many cases - third and second ones. In this connection $N(p)$ and $D(p)$ for a case of the sixth order are presented as follows

$$N_{n=6}(p) = F_{rp} F_{rv} F_{acl} \Omega_1 \Omega_3 \Omega_5 \Omega_7 \Omega_9 / (J_6 p^2);$$

$$D_{n=6}(p) = 1 + \alpha_1 + \alpha_2 + \dots + \alpha_5,$$

where

$$\begin{aligned} \Omega_1 &= (\omega_{01}/p)^2, \Omega_2 = (\omega_{02}/p)^2, \Omega_3 = (\omega_{03}/p)^2, \Omega_4 = (\omega_{04}/p)^2, \Omega_5 = (\omega_{05}/p)^2, \\ \Omega_6 &= (\omega_{06}/p)^2, \Omega_7 = (\omega_{07}/p)^2, \Omega_8 = (\omega_{08}/p)^2, \Omega_9 = (\omega_{09}/p)^2, \Omega_{10} = (\omega_{010}/p)^2, \\ \alpha_1 &= \Omega_1 + \Omega_2 + \dots + \Omega_{10} + \frac{F_{rp}F_{rv}F_{acl}}{J_6p^2}, \\ \alpha_2 &= \Omega_1(\Omega_3 + \dots + \Omega_{10}) + \Omega_2(\Omega_4 + \dots + \Omega_{10}) + \Omega_3(\Omega_5 + \dots + \Omega_{10}) + \Omega_8\Omega_{10}, \dots, \\ \alpha_5 &= \Omega_1\Omega_3\Omega_5\Omega_7(\Omega_9 + \Omega_{10}) + \Omega_1\Omega_3\Omega_6\Omega_8\Omega_{10} + \Omega_2\Omega_4\Omega_6\Omega_8\Omega_{10}. \end{aligned}$$

At a vector of coordinates x_{k1} , if to conclude the added members from loops (summands introduced by the account of negative feedbacks of velocities of masses) in brackets of a kind $\{\}$, expressions for $N(p)$ and $D(p)$ in (2) can be presented as

$$\begin{aligned} N_{n=6}^f(p) &= F_{rp}F_{rv}F_{acl}\Omega_1\Omega_3\Omega_5\Omega_7\Omega_9/(J_6p^2), \\ D_{n=6}^f(p) &= 1 + \alpha_1 + \{\Omega_{\omega_1}(1 + \Omega_2 + \Omega_2\Omega_4 + \Omega_2\Omega_4\Omega_6 + \Omega_2\Omega_4\Omega_6\Omega_8 + \Omega_2\Omega_4\Omega_6\Omega_8\Omega_{10})\} + \alpha_2 + \rightarrow \\ &\rightarrow + \{\Omega_{\omega_1}(\Omega_2 + \dots + \Omega_{10}) + \Omega_{\omega_2}(\Omega_4 + \dots + \Omega_{10}) + \Omega_{\omega_3}(\Omega_6 + \dots + \Omega_{10}) + \Omega_{\omega_4}(\Omega_8 + \Omega_9 + \Omega_{10}) + \rightarrow \\ &\rightarrow + \Omega_{\omega_5}\Omega_{10}\} + \dots + \alpha_5 + \{\Omega_{\omega_1}\Omega_2\Omega_4\Omega_6(\Omega_8 + \Omega_9 + \Omega_{10}) + \Omega_{\omega_1}\Omega_2\Omega_4\Omega_7(\Omega_9 + \Omega_{10}) + \rightarrow \\ &\rightarrow + \Omega_{\omega_1}\Omega_2\Omega_4\Omega_8\Omega_{10} + \dots + \Omega_{\omega_1}\Omega_4\Omega_6\Omega_8\Omega_{10} + \Omega_{\omega_2}\Omega_4\Omega_6\Omega_8\Omega_{10}\} + \{\Omega_{\omega_1}\Omega_2\Omega_4\Omega_6\Omega_8\Omega_{10}\}, \end{aligned}$$

where

$$\begin{aligned} \Omega_{\omega_1} &= F_{rv}F_{acl}/(J_1p), \Omega_{\omega_2} = \Omega_2\Omega_{\omega_1}, \Omega_{\omega_3} = \Omega_4\Omega_{\omega_2} = \Omega_2\Omega_4\Omega_{\omega_1}, \Omega_{\omega_4} = \Omega_6\Omega_{\omega_3} = \Omega_2\Omega_4\Omega_6\Omega_{\omega_1}, \\ \Omega_{\omega_6} &= \Omega_8\Omega_{\omega_4} = \Omega_2\Omega_4\Omega_6\Omega_8\Omega_{\omega_1}, \Omega_{\omega_8} = \Omega_1\Omega_{\omega_6} = \Omega_2\Omega_4\Omega_6\Omega_8\Omega_1\Omega_{\omega_1}. \end{aligned}$$

Then the characteristic equation can be written down as

$$D^{n=6}(p) = a_0p^{12} + a_1p^{10} + a_2p^8 + a_3p^6 + a_4p^4 + a_5p^2 + a_6, \quad (5)$$

where

$$\begin{aligned} a_0 &= \chi_6/\theta, \quad a_1 = \chi_6[\omega_{01}^2 + \dots + \omega_{010}^2]/\theta, \\ a_2 &= \chi_6[\omega_{01}^2(\omega_{03}^2 + \dots + \omega_{010}^2) + \omega_{02}^2(\omega_{04}^2 + \dots + \omega_{010}^2) + \dots + \omega_{08}^2\omega_{010}^2]/\theta, \\ a_3 &= \chi_6[\omega_{01}^2\omega_{03}^2(\omega_{05}^2 + \dots + \omega_{010}^2) + \omega_{01}^2\omega_{04}^2(\omega_{06}^2 + \dots + \omega_{010}^2) + \dots + \omega_{01}^2\omega_{07}^2(\omega_{09}^2 + \omega_{010}^2) + \omega_{01}^2\omega_{08}^2\omega_{010}^2 + \rightarrow \\ &\rightarrow + \omega_{02}^2\omega_{04}^2(\omega_{06}^2 + \dots + \omega_{010}^2) + \dots + \omega_{02}^2\omega_{08}^2\omega_{010}^2 + \dots + \omega_{05}^2\omega_{07}^2(\omega_{09}^2 + \omega_{010}^2) + \omega_{05}^2\omega_{08}^2\omega_{010}^2 + \omega_{06}^2\omega_{08}^2\omega_{010}^2]/\theta \end{aligned}$$

$$a_4 = \chi_6 [\omega_{01}^2 \omega_{03}^2 \omega_{05}^2 (\omega_{07}^2 + \dots + \omega_{010}^2) + \dots + \omega_{01}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2 + \omega_{02}^2 \omega_{04}^2 \omega_{06}^2 (\omega_{08}^2 + \omega_{09}^2 + \omega_{010}^2) + \dots + \rightarrow \\ \rightarrow + \omega_{02}^2 \omega_{04}^2 \omega_{08}^2 \omega_{010}^2 + \dots + \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2] / \theta,$$

$$a_5 = \chi_6 [\omega_{01}^2 \omega_{03}^2 \omega_{05}^2 \omega_{07}^2 (\omega_{09}^2 + \omega_{010}^2) + \omega_{01}^2 \omega_{03}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2 + \omega_{01}^2 \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2 + \omega_{02}^2 \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2] / \theta,$$

$$a_6 = 1, \theta = F_{rv} F_{acl}, \chi_6 = J_6 / [F_{rp} (\omega_{01} \omega_{03} \omega_{05} \dots \omega_{09})^2].$$

At a vector of coordinates x_{k1} , if to omit the intermediate calculations, the characteristic equation (5) at $n=6$ can be presented by mathematical expression

$$D_{x_{k1}}^{n=6}(p) = b_0 p^{12} + b_1 p^{11} + b_2 p^{10} + \dots + b_{10} p^2 + b_{11} p + b_{12} = 0, \quad (6)$$

where

$$b_0 = a_0 = \chi_6 / \theta, \quad b_1 = \chi_6 / J_1, \quad b_2 = a_1 = \chi_6 [\omega_{01}^2 + \dots + \omega_{010}^2] / \theta, \dots,$$

$$b_{10} = a_5 = \chi_6 [\omega_{01}^2 \omega_{03}^2 \omega_{05}^2 \omega_{07}^2 (\omega_{09}^2 + \omega_{010}^2) + \omega_{01}^2 \omega_{03}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2 + \omega_{01}^2 \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2 + \rightarrow \\ \rightarrow + \omega_{02}^2 \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2] / \theta,$$

$$b_{11} = \chi_6 [5 \omega_{02}^2 \omega_{04}^2 \omega_{06}^2 \omega_{08}^2 \omega_{010}^2] / J_1, \quad b_{12} = a_6 = 1.$$

Then conditions' system of stability for $n=6$, at the found values of coefficients b_0, \dots, b_{11} , has a form stated below

$$\left. \begin{aligned} b_0 > 0, b_1 > 0, b_2 > 0, b_3 > 0, \dots, b_i > 0, \\ C_{ii} = b_{i+1} - b_{i+2r_i} > 0, \end{aligned} \right\} \quad (7)$$

where $r_i = b_{i-1} / b_i$, $i=1, \dots, 2_{n-2}$ ($n=6$).

Hence at the three-mass presentation of a mechanical part of EMS these conditions can be presented as

$$\left. \begin{aligned} d_0 &= \chi_3 / \theta = J_3 / [(F_{rp} \omega_{01}^2 \omega_{03}^2) \theta] > 0, \\ d_1 &= \chi_3 / J_1 > 0, \\ d_2 &= \chi_3 (\omega_{01}^2 + \omega_{02}^2 + \omega_{03}^2 + \omega_{04}^2) / \theta > 0, \\ d_3 &= \chi_3 (\omega_{02}^2 + \omega_{03}^2 + \omega_{04}^2) / J_1 > 0, \\ d_4 &= \chi_3 [\omega_{01}^2 (\omega_{03}^2 + \omega_{04}^2) + \omega_{02}^2 \omega_{04}^2] / \theta > 0, \\ d_5 &= \chi_3 \omega_{02}^2 (1 + \omega_{04}^2) / J_1 > 0, \\ D_1 &= d_1 d_2 - d_3 d_0 > 0, \\ D_2 &= d_3 D_1 - d_1 (d_4 d_1 - d_5 d_0) > 0, \\ D_3 &= (d_4 d_1 - d_5 d_0) D_2 + (d_1 d_2 - d_3 d_0) (d_5 D_1 - d_6 d_1^2) > 0, \\ D_4 &= (d_5 D_1 - d_6 d_1^2) D_3 - d_1 D_2^2 > 0, \end{aligned} \right\} \quad (8)$$

where d_0, \dots, d_6 – coefficients of the characteristic equation

$$D(p) = d_0 p^6 + d_1 p^5 + d_2 p^4 + d_3 p^3 + d_4 p^2 + d_5 p + d_6 = 0 .$$

In the case of the two-mass performance of a mechanical part of EMS these conditions are become

$$\left. \begin{aligned} l_0 &= \chi_2 / \theta = J_2 / [(F_{rp} \omega_{01}^2) \theta] > 0, \\ l_1 &= \chi_2 / J_1 > 0, \\ l_2 &= \chi_2 (\omega_{01}^2 + \omega_{02}^2) / \theta > 0, \\ l_3 &= 2 \chi_2 \omega_{02}^2 / J_1 > 0, \\ l_1 l_2 - l_3 l_0 &> 0, \\ l_3 (l_1 l_2 - l_3 l_0) - l_4 l_1^2 &> 0, \end{aligned} \right\} (9)$$

where l_1, l_2, l_3 - coefficients of the characteristic equation

$$L(p) = l_0 p^4 + l_1 p^3 + l_2 p^2 + l_3 p + l_0 = 0 .$$

The found conditions (7) - (9) at $(n =) 6, 3, 2$ - mass elastic objects and cascade control enable to check stability of systems developed by a choice of structure and parameters of regulators of a position ($F_{rp}(p)$), velocity ($F_{rv}(p)$), and closed armature current loop ($F_{aci}(p)$) determined, first of all, by structure and element basis of a power part of EMS. The similar consideration is simple for case study of systems with different of the number of the cascade loops.

At the present time systems of the cascade control for complex multi-mass elastic EMS are synthesized from the point of view of selection of these structures and calculation of their parameters. However, even in the best way, in the most cases synthesized structure and the parameters of control systems of EMS deduce it on frontier areas of stability. As a result there are states of nonoperability and instability of its adjusting characteristics.

The advantage of the given approach for verification of system stability of a cascade regulation of EMS consists in the account of multi-mass and elastic links of a mechanical part and, as a consequence, - the opportunity of construction of control regulation in such a way as to exclude influence of intermass resonant effects on its stability as a whole. It is a critical point for consideration of EMS with low damper properties of the electric drive (and its passband) in ranges of its functioning and intermass interaction.

For example, designing and realizing of β -test mechanisms of the parabolic antenna with a 5-meter diameter at an azimuth tracking control system as three-mass elastic EMS one can see that with the certain degree of simplification such EMS is presented by three allocated elastic basics connected among themselves masses (rotor of the engine, worm-and-wheel gearbox, and lead pinion gear, a platform on which

is placed an elevation drive of the parabolic antenna). Research of mechanical part of such system pointed out that its mechanical vibrations were arose because of basic interaction of the second and third masses and as a consequence of it – increasing of mechanical overloads and underestimating real dynamic loads in mechanical links of executive mechanism on the basis of significant error of classic mathematical approach of calculation.

Complicated research depicted that if the given system is considered as two-mass elastic EMS

- with the moments of inertia of the engine - $J_1 = 0.41 \text{ кг} \cdot \text{м}^2$ and worm-and wheel gear, lead pinion gear, a platform - $J_2 = 0.36 \text{ кг} \cdot \text{м}^2$,
- with rigidity of mechanical links between the first and second masses - $C_{12} = 5180 \text{ Н} \cdot \text{м}$,
- with gear-ratio – $R = 366$ and $F_{acl} = 23$ at coefficient of amplification of power converter $k_c = V_c / V_{in} = 208 / 10 = 20.8$ where $V_c = 208 \text{ В}$ - voltage of power supply, $V_{in} = 10 \text{ В}$ - maximal input given voltage on the closed current loop, $F_{rv} = 133$, and $F_{rp} = 74$,

the system has frequencies which equal $\omega_{01} = 112.4 \text{ Hz}$, $\omega_{02} = 84.8 \text{ Hz}$, and $\theta = F_{rp} \cdot F_{acl} = 2599$, $\chi_2 = J_2 / (F_{rp} \cdot \omega_{01}^2) = 0.36 / (74 \cdot 112.4) = 0.432 \cdot 10^{-4} \text{ кг} \cdot \text{м}^2 \cdot \text{с}^2$. Moreover the last two conditions of system (9) are carried out – $5442.7 > 0$ and $0.3 > 0$ accordingly. In spite of the last condition is carried out one can see that at a slight deviation from design values the condition can be not executed. Besides exacter ratings of system stability and elimination of the reasons of vibrations in the designed system were given on the basis of conditions (8) at which the system was represented as three-mass with $J_1 = 0.41 \text{ кг} \cdot \text{м}^2$, $J_2^* = 0.1 \text{ кг} \cdot \text{м}^2$, $J_3^* = 0.26 \text{ кг} \cdot \text{м}^2$ ($J_2 = J_2^* \cdot J_3^* = 0.36 \text{ кг} \cdot \text{м}^2$), $c_{12}^* = 45.2 \cdot 10^4 \text{ Н} \cdot \text{м}$, $c_{23}^* = 5240 \text{ Н} \cdot \text{м}$ / $c_{12} = (c_{12}^* + c_{23}^*) / (c_{12}^* \cdot c_{23}^*) = 5180 \text{ Н} \cdot \text{м}$ /, $i_2 = 59$, $i_3 = 6.2$ ($i = i_2 \cdot i_3 = 366$), $F_{acl} = 23$, $F_{rv} = 133$, $F_{rp} = 74$. In addition the own frequencies of EMS are $\omega_1^* = 1050 \text{ Hz}$, $\omega_{02}^* = 21.3 \cdot 10^2 \text{ Hz}$, $\omega_{03}^* = 228.9 \text{ Hz}$, $\omega_{04}^* = 141.9 \text{ Hz}$. The check of system conditions (8) showed that the last condition is not carried out. Furthermore the analysis displayed that for carrying out conditions (8), without change of a mechanical design and its parameters, it is more expedient to reduce F_{rv} ($F_{rv} = 95$). Let me note that at such correction the last two conditions of system (9) also are carried out (with the large margin) – as $5442.7 > 0$ and $0.55 > 0$ accordingly.

Thus, it is expedient to check carrying out of the found conditions of stability on design stages and adjustment of cascade control systems for complex multi-mass elastic EMS.

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УСЛОВИЯ СИСТЕМНОЙ СТАБИЛЬНОСТИ ПРИ КАСКАДНОМ УПРАВЛЕНИИ
ЭЛЕКТРОМЕХАНИЧЕСКОЙ СИСТЕМЫ С МНОГОМАССОВОЙ УПРУГОЙ
МЕХАНИЧЕСКОЙ ЧАСТЬЮ

Рассмотрены условия устойчивости при каскадном способе управления для электромеханической системы (ЭМС) с упругой механической частью на базе принципов структурных преобразования Мезона, алгебраического критерия устойчивости Рауса-Найквиста и принципов подчиненного регулирования. Получена система условий выполнения которых позволяет выполнить ЭМС стабильной в любых режимах её работы и во всем диапазоне рабочих характеристик.